Improved Reduction Rules, Implemented in Peter's Engine

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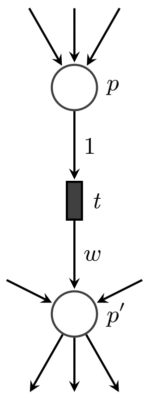
The rules are presented in Figures 1 and 2 and they are relative to a given initial marking M_0 and a cardinality query φ, where $places(\varphi)$ is the set of all places that occur in the query φ.

Theorem 1. *Let (N, M_0) be a marked Petri net and let φ be a cardinality query. Let N' be the net N after the application of some reduction rules from Figures 1 and 2. Then $(N, M_0) \models EF\, \varphi$ if and only if $(N', M_0) \models EF\, \varphi$.*

Theorem 2. *Let (N, M_0) be a marked Petri net. Let N' be the net N after the application of some reduction rules from Figures 1 and 2 for a query $\varphi = 2 < 1$. Then (N, M_0) has a deadlock if and only if (N', M_0) has a deadlock.*

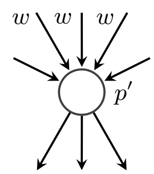
For the inhibitor-arc, we use $I(p, t) \in \mathbb{N} \cup \{0\}$. As a shorthand we write $I(p) = \{t \mid t \in T \text{ and } I(p, t) \neq 0\}$ (and $I(t) = \{p \mid p \in P \text{ and } I(p, t) \neq 0\}$) to denote the set of transitions (or places) which are connected via inhibitor-arcs.



Conditions on p, t and p':

A1: $p \neq p'$
A2: $p^\bullet = \{t\}$, ${}^\bullet t = \{p\}$, $t^\bullet = \{p'\}$
A3: $F(p,t) = 1$ and $w = F(t,p') \geq 1$
A4: $I(t) = I(p) = I(p') = \emptyset$
A5: $p, p' \notin places(\varphi)$

$\Rightarrow$

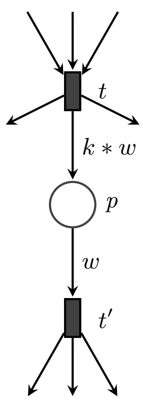


UA1: Remove p and t.

UA2: $M_0(p') := M_0(p') + M_0(p) * w$.

For all $t' \in T \setminus \{t\}$:
UA3: $F(t',p') := w * F(t',p) + F(t',p')$

(a) Rule A: Sequential transition removal

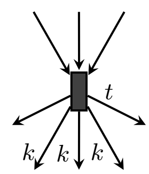


Conditions on t, p and t':

B1: $t \neq t'$
B2: ${}^\bullet p = \{t\}$, $p^\bullet = \{t'\}$, ${}^\bullet t' = \{p\}$
B3: $F(p,t') = w \geq 1$ and $F(t,p) = k * w$ where $k \geq 1$
B4: $I(p) = I(t) = I(t') = \emptyset$
B5: $p \notin places(\varphi)$

for all $p' \in t'^\bullet$:
B6: $I(p') = \emptyset$
B7: $p' \notin places(\varphi)$

$\Rightarrow$

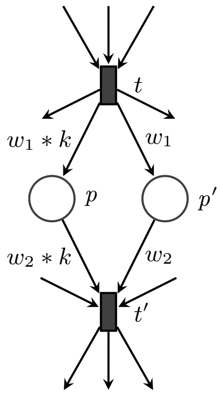


UB1: Remove p and t'.

For all $p' \in P$:
UB2: $M_0(p') = M_0(p') + \lfloor M_0(p)/w \rfloor * F(t',p')$
UB3: $F(t,p') := F(t,p') + k * F(t',p')$

(b) Rule B: Sequential place removal

Fig. 1: Sequential rules for a cardinality formula φ and initial marking M_0



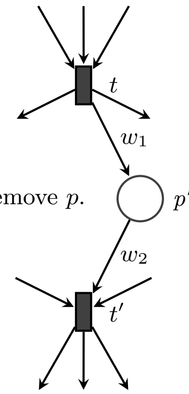
Conditions on t, t', p and p':

C1: $p \neq p'$
C2: ${}^{\bullet}p = {}^{\bullet}p'$
C3: $p^{\bullet} = p'^{\bullet}$
C4: $I(p) = I(p') = \emptyset$
C5: $p \notin \mathit{places}(\varphi)$

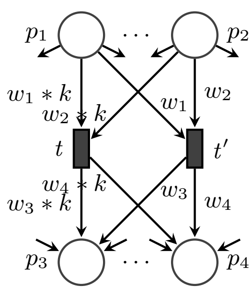
There exists a $k > 0$ s.t.

C6: $M_0(p) \geq M_0(p') * k$
C7: $\forall t \in {}^{\bullet}p.\ F(t, p) = F(t, p') * k$
C8: $\forall t' \in p^{\bullet}.\ F(p, t') = F(p', t') * k$

$\Rightarrow$ UC1: Remove p.



(a) Rule C: Parallel place removal



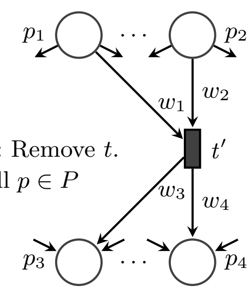
Conditions on t and t':

D1: $t \neq t'$
D2: $I(t) = I(t') = \emptyset$

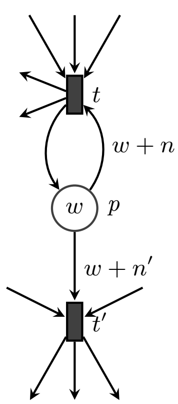
$\Rightarrow$ UD1: Remove t.

There exists a $k > 0$ s.t. for all $p \in P$

D3: $F(p, t) = F(p, t') * k$
D4: $F(t, p) = F(t', p) * k$



(b) Rule D: Parallel transition removal



Conditions on p, t and p':

A1: ${}^{\bullet}p \subseteq p^{\bullet}$
A2: $I(p) = \emptyset$
A3: $p \notin \mathit{places}(\varphi)$
A4: for all $t \in p^{\bullet}$ it holds that $M_0(p) < F(p, t)$.

UE1: Remove p
UE2: Remove all $t \in p^{\bullet}$

(c) Rule E: Dead place removal

Fig. 2: Parallel rules for a cardinality formula φ and initial marking M_0